Instability in the hydrostatic extrusion of polymers

With the increasing use of polymers in engineering, there has been much interest in their methods of production and the development of materials having improved physical properties, particularly in the case of sections. Although all commercial extrusions are produced at temperatures well above their melting point, interest is being shown in solid phase forming, notably of linear polyethylenes, where a large amount of deformation results in a highly oriented product with a much increased axial stiffness [1]. One of the more interesting techniques being applied is that of hydrostatic extrusion where the pressure of a fluid surrounding the billet is used to force it through a die. This results in a reduction in the work done and an improvement in the lubrication conditions in the deformation zone when compared with conventional extrusion. However, as in the case of the hydrostatic extrusion of metals, the process is not always stable, since any increase in the rate of extrusion results in improved lubrication and, in the case of polymers, an increase in temperature (due to their poor thermal conductivity) causing softening. Thus the extrusion, once started, gets progressively faster unless the pressure required for the increased rate of working balances out the velocity dependent reduction [2]. In view of this, it has been found advantageous to use an augmenting force, so that only about 90% of the required energy is derived from the hydrostatic pressure, the rest being supplied by a controlled force applied to the product or billet [3]. Product augmentation has been used in the hydrostatic extrusion of polymers to help provide stable conditions and the conclusion has been drawn that such a system should use velocity rather than force, as the controlled parameter so that a reduction in draw force occurs automatically should the material leave the die at a greater velocity than that at which it is being drawn (Fig. 1). Unfortunately, the instabilities occurring in the hydrostatic extrusion of polymers cannot always be dealt with in this manner since the extruded product or tag used to apply the draw force, itself becomes part of the system and its own dynamic characteristics have to be taken into account. Consequently, large oscillations of the draw force may occur during constant velcoity augmentation, even to the point of the draw force dropping to zero and the product being expelled violently from the die. Such oscillation has been found to occur with the longer lengths of tag; the damping ratio of the system appearing to decrease as the extrusion progresses.

It was against this background and concurrent with a programme of research into the production of high stiffness polymers, that a project was undertaken to develop a control system suitable for the hydrostatic extrusion of polymers so that sections could be produced under stable conditions and to a reproducible specification. The work outlined here was a further development to produce a generalized dynamic model so that the instability experienced may be explained, and so that the knowledge gained with one particular polymer may be more easily adapted to a range of similar materials. The materials used in the practical tests have been low density polyethylene (Alkathene) and Nylon 6,6, although the theory being developed should apply to a wide range of polymers.

In determining the dynamic characteristics of a system, it is necessary to provide an input whilst measuring the change in output, the simple input alternatives being impulse, step ramp and sine-wave functions. However, in this case the obvious form of the input, a change in draw force, results in the

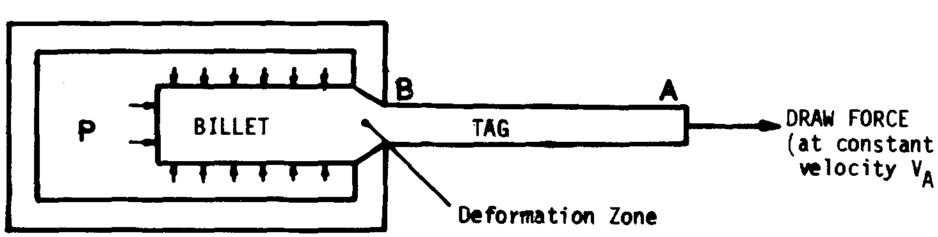

Figure 1 Product augmented hydrostatic extrusion.

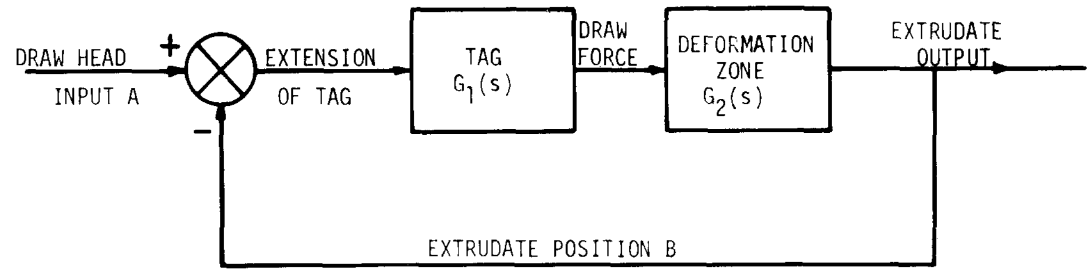

Figure 2 Hydrostatic extrusion shown as a feedback system.

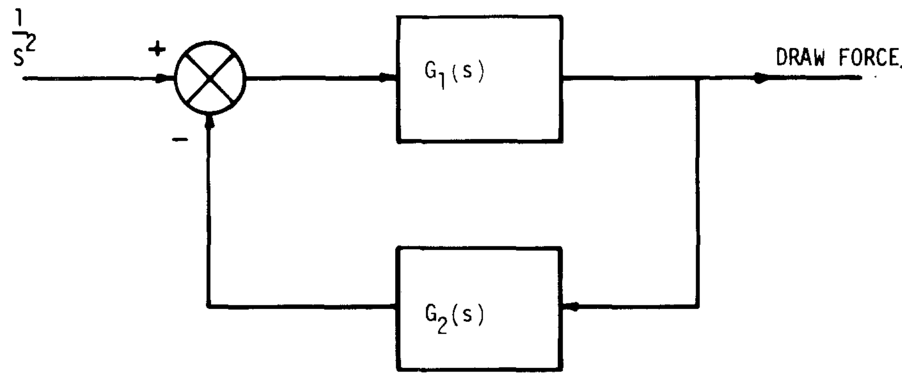

Figure 3 Hydrostatic extrusion shown with draw force as the output.

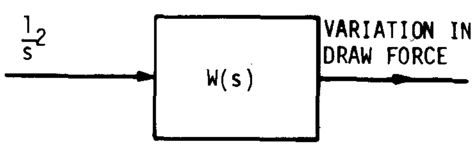

Figure 4 Reduction of Fig. 3.

extrusion system departing from its steady state too rapidly for a useful result to be obtained. An alternative input is a change of velocity in a step manner, for example a sudden change from 2 mm sec^{-1} to 2.5 mm sec^{-1}, both velocities being known to provide stable, reproducible extrusion conditions for the range of tests previously performed. At this stage the material deformation process is studied in order to arrive at a model from which it is possible to attach significance to a change in input velocity.

From Fig. 1 it can be seen that the material in the deformation zone is subjected to a stress arising from: (1) a constant high hydrostatic pressure, P, (2) a draw force transmitted through the tag of extruded product.

This drawing force will only be constant if the velocity at which the material leaves the die (V_B) is equal to the velocity of the drawing head (V_A). Any difference in these velocities results in an extension of the tag leading to a *variable* draw force at the deformation zone.

The extrusion system in Fig. 2, using control engineering terminology, shows the extension of the tag as the real input to the system. Thus, the extension is modified by the transfer function of the tag, $G_1(s)$, into a force which acts on the deformation zone, $G_2(s)$, producing a related extrusion. The effect of hydrostatic pressure may be neglected since it is held constant. At this point it is of interest to note that even if the velocity at A equals that a B the force in the tag will decrease owing to the viscoelasticity of polymers.

Unfortunately, the extrusion must take place at low velocities so that softening due to a rise in temperature of the product may be neglected, and it is virtually impossible to measure the actual extrusion velocity with any accuracy. The draw force, however, can be measured from a load cell on the drawing head and if the block diagram is rearranged, this becomes the output of the system. The deformation zone is now in the feedback path as shown in Fig. 3. The input change in velocity is shown in Laplace terms as $1/s^2$, and the system further reduces to that shown in Fig. 4.

Tests on extruded material yield an approximate transfer function for the tag of the form:

$$G_1(s) = \frac{K_1\,(s+a)}{(s+b)}$$

where K_1 is proportional to the tensile modulus.

Thus, the application of an instantaneous extension of 1 mm results in a force in the tag initially of magnitude K_1 decaying exponentially to a steady state value of $K_1 \times a/(b)$. (Although this is a very simple model it holds well over the first few seconds after the application of a step strain input.)

It may be seen that as extrusion progresses the tag length increases and the value of K_1 decreases. In contrast, it may be conjectured that the transfer function for the deformation zone will not change as extrusion progresses and if it can be found, then $W(s)$, the overall transfer function, may be determined from:

$$W(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}.$$

It is clear that the behaviour of the extrusion process depends on $W(s)$ and in turn on the length of tag, so that it should be possible to proceed with the analysis and predict the length of tag (or value of K_1) that causes instability, or alternatively, the position of the roots of $W(s)$ for any value of K_1. This is done using the Root Locus Method.

In practice, $G_2(s)$ is found using $G_1(s)$ and the response of $W(s)$ when the draw velocity is suddenly increased. A typical result for the transient response of the system for an intermediate length of tag is shown in Fig. 5, from which it is seen that an increase in velocity gives a decrease in the steady state draw force required. Analysis of transient response data for a particular set of conditions gives a result of the form:

$$W(s) = \frac{K_2\,(s-c)}{s^2 + ds + e}.$$

The results obtained to date being good linear approximations to what is almost certainly a nonlinear system. However, proceeding with the assumption of linearity gives:

$$G_2(s) = \frac{K_3\,(s-g)\,(s+h)\,(s+k)}{s(s-m)\,(s+n)}.$$

(The pole at $+m$ shows the deformation zone to have the expected unstable response.) The expressions for $G_1(s)$ and $G_2(s)$ may now be used to plot a root locus diagram for $\infty > K_1 > 0$, that is for extrusion with the tag length varying from zero to infinity. This is shown in Fig. 6.

For the limited range of results obained to date there is good agreement with the theoretical root locus for small values of K_1, the system becoming progressively less stable as the length of tag increases. The root locus indicates that instability will be encountered at some tag length for any viscoelastic material undergoing product augmented

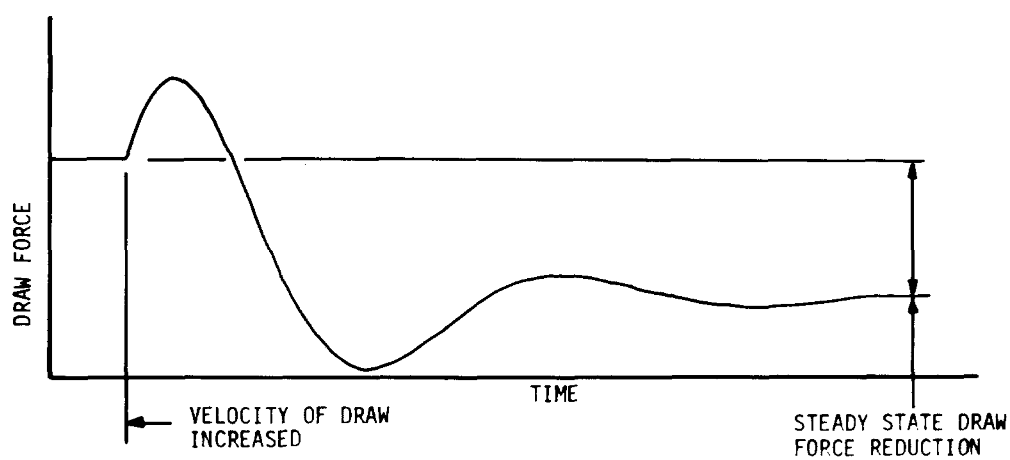

Figure 5 Dynamic variation in draw force due to a change in draw velocity. (Transient response of $W(s)$ to an input of $1/s^2$.)

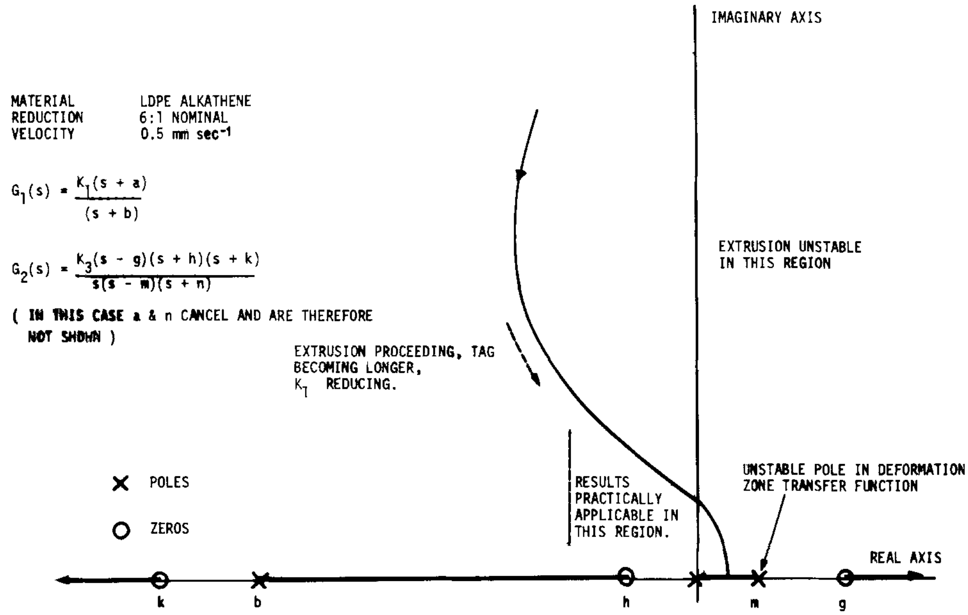

Figure 6 Theoretical root locus for the hydrostatic extrusion of low density polyethylene.

hydrostatic extrusion, since different material characteristics will merely change the magnitude of the roots of $G_1(s)$ and $G_2(s)$, not their sign. However, the root locus for large values of K_1, or a very short tag, is not correct, and it seems that the expression for $G_2(s)$ is not the correct one. Work is proceeding on the analogue computer modelling of the system assuming that the deformation zone, $G_2(s)$, is a fairly simple unstable system with the addition of a dead band. Thus although unstable, there is a minimum increment of force necessary to overcome certain fixed parameters in the deformation zone. Preliminary results from this kind of model are encouraging and the assumption of non-linearity would seem to be reasonable. However, with all the models considered to date it has become clear that instability will occur with long tag lengths if the deformation zone is unstable to a force input. Thus although the process of hydrostatic extrusion of polymers is of great interest, it would seem from an initial investigation that its stability is suspect when using product augmentation.

Although this work is applied to hydrostatic extrusion, it is clearly applicable to any processing of viscoelastic material where the deformation zone is remote from the point of application of the force. The use of a "feedback" analogy may help in the explanation of other dynamic phenomena in materials processing.

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